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B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2024
THIRD YEAR (BATCH 2022-25)
MATHEMATICS (HONOURS)
Paper : CC 12

Date : 03/12/2024
Time : 11.00 am – 1.00 pm
Full Marks : 50

Group - A

Answer all questions. Maximum you can obtain is 40.

1. Find the equation of the tangent plane to
$z = 3y^2 - 2x^2 + x$ at (2,-1,-3). [2]

2. Find the maximum rate of change of $f(x,y) = \sqrt{x^2 + y^4}$ at (-2,3) and the direction in which this maximum rate of change occurs. [2]

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. If $g(u,v) = f(u^2 - v^2)$, then prove that $g_{uu} + g_{vv} = 4(u^2 + v^2) f^{//}(u^2 - v^2)$. [3]

4. If $S \subseteq \mathbb{R}^2$, let $f : S \to \mathbb{R}^2$ be a function with values in $\mathbb{R}^2$, and write $f = (f_1, f_2)$. Prove that f is differentiable at an interior point c of S if and only if, each $f_i (i = 1,2)$ is differentiable at c. [4]

5. Prove or disprove : let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function. If all the directional derivatives of f exist at a point $p \in \mathbb{R}^2$, then f is differentiable at p. [3]

6. Find the local maximum and minimum values and saddle point(s) of the function:
$f(x,y) = y^3 + 3x^2 y - 6x^2 - 6y^2 + 2$. [5]

7. If $V = cos^{-1} \dfrac{x+y}{\sqrt{x} + \sqrt{y}}$, then verify that $cos V = \dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ is a homogeneous function of x,y of degree $\frac{1}{2}$. Hence prove $x\dfrac{\partial V}{\partial x} + y\dfrac{\partial V}{\partial y} + \dfrac{1}{2} cot V = 0$. [3]

8. State implicit function theorem for two variables. Prove that in a neighbourhood of the point $\left(\sqrt{e}, \dfrac{1}{\sqrt{e}}\right)$ the curve defined by the equation $xy = log\dfrac{x}{y}$ is the graph of a function $y = f(x)$. [2+2]

9. Using Lagrange's multiplier method find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest and farthest from the point (3,1, -1). [5]

10. Evaluate the integral $\iint_R \sqrt{x^2 + y^2}\, dxdy$ by changing to polar coordinates, where R is the region in the x-y plane bounded by the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$. [3]

11. Evaluate $\displaystyle\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2}\, dxdy$ applying the transformation $u = \dfrac{2x-y}{2}, v = \dfrac{y}{2}$ and integrating over an appropriate region in the uv plane. [5]

12. Find the work done by the force field $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ in moving an object along the curve C parameterized by $r(t) = cos(\pi t)\hat{i} + t^2\hat{j} + sin(\pi t)\hat{k},\ 0 \le t \le 1$. [3]

13. Show that $\vec{F} = (2x-3)\hat{i} - z\hat{j} + (cos\, z)\hat{k}$ is not conservative. [2]

14. Find the flux of $\vec{F} = xy\hat{i} + yz\hat{j} + xz\hat{k}$ outward through the surface of the cube cut from the first octant by the planes $x = 1, y = 1,$ and $z = 1$. [4]

Group - B

Answer *any two* questions. All the notations have their usual meaning.

15. Expand $f(x) = x^2$ as a Fourier series in the interval $(-\pi, \pi)$ and hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... = \frac{\pi^2}{6}$. [4+1]

16. Obtain the Fourier series for $f(x) = x$ in $(-\pi, \pi)$ and prove that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{4}$. [4+1]

17. Expand $f(x) = |x|$ as a Fourier series in the interval $(-\pi, \pi)$ and hence deduce that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8} \ .$$ [4+1]

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